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KS5 Mathematics induction booklet.

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| **Essential Pre-Knowledge Tasks**  **(You will be tested on these soon after the start of term)** |
| The A level course builds on the GCSE course and we will assume that you have a sound grasp of the skills from your GCSE. We also know that maths can go rusty very quickly if you do not practice it. With that in mind it is essential that you do some practice over the summer in order to get off to the best start possible on the A level course. There are three elements to the preparation for September.  Task 1: Algebra Practice  Please ensure you complete the tasks at the end of this pack in preparation for September.  Please see the practice sheets below on   * Expanding Brackets * Surds * Indices * Factorising   Complete these tasks, and mark your work. Make sure you do corrections.  **You will have an induction test in your first week based on this work.** |
| Task 2: AMSP Website  [Transition to A level Mathematics resources: Essential Skills | AMSP](https://amsp.org.uk/resource/gcse-alevel-transition-resources)  The above website has some excellent resources to refresh and stretch your GCSE knowledge. You may want to use it in conjunction with the above task as there are some help videos and explanations you may find helpful.  There are 6 sections:   * Simplifying * Expanding * Factorising * Rearranging * Solving * Sketching   Each has 3 sections in, and each section is estimated to take about an hour. You may find that it is quicker, if you can remember the skills! The tasks range from routine practice through to some more challenging problems to solve. Please use the website to help recap, practice and inspire you. Have fun with the problems! |

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| **Book Recommendations**  It is vital that you are accustomed to completing wider reading around topics you will cover during your A levels. As a starting point, we recommend the following titles: |
| http://ecx.images-amazon.com/images/I/51Lh6F29sLL._SX329_BO1,204,203,200_.jpg **Alex’s Adventures in Numberland – Alex Bellos**  Exploding the myth that maths is best left to the geeks, Alex Bellos covers subjects from adding to algebra, from set theory to statistics and from logarithms to logical paradoxes. In doing so, he explains how mathematical ideas underpin just about everything in our lives.  https://images-na.ssl-images-amazon.com/images/I/719yLXSSqIL.jpg  **Professor Stewart’s Hoard of Mathematical Treasures – Ian Stewart**  Ian Stewart presents a new and magical mix of games, puzzles, paradoxes, brainteasers, and riddles. He mingles these with forays into ancient and modern mathematical thought, appallingly hilarious mathematical jokes, and enquiries into the great mathematical challenges of the present and past  https://images-na.ssl-images-amazon.com/images/I/51y08BY1mKL.jpg**The Indisputable Existence of Santa Claus – Dr Hannah Fry and Dr Thomas Oleron Evans**  Full of diagrams, sketches and graphs, beautiful equations, Markov chains and matrices, *The Indisputable Existence of Santa Exists* brightens up the bleak midwinter with stockingfuls of mathematical marvels. And proves once and for all that maths isn't just for old men with white hair and beards who associate with elves. |

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| **Film/ Documentary Recommendations**  There are a number of useful films and documentaries that will develop your wider understanding of the topics covered. |
| Image result for hidden figures film**Hidden Figures - PG**  The story of a team of female African-American mathematicians who served a vital role in NASA during the early years of the U.S. space program.  Image result for beautiful mind  **Beautiful Mind - 12**  After John Nash, a brilliant but asocial mathematician, accepts secret work in cryptography, his life takes a turn for the nightmarish.  Image result for the imitation game  **The Imitation Game – 12**  Biopic of Alan Turing, the brilliant mathematician and computer scientist who helped the Allies secure victory in World War II by cracking the German Enigma code, and who was later prosecuted as a homosexual by his own government.  Any program with Marcus du Sautoy!  - Radio: BBC radio 4 – A Brief History of Mathematics <http://www.bbc.co.uk/programmes/b00srz5b> |

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| **Enrichment Activity** |
| Download the free sumaze! apps. No instructions, learn by playing. Once you have completed the logarithms levels, research logarithms online. Can you find the unknown values below?    log2(32) = x x = ?  logy(243) = 5 y = ?  log7(z) = 5 z = ? |

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| **Ideas for Day Trips**  Visiting some of the places in the list below could be fun AND educational…. |
| **The science Museum-** From war and peace to life, death, money, trade and beauty, the objects in Mathematics: The Winton Gallery reveal how mathematics connects to every aspect of our lives.  <https://www.sciencemuseum.org.uk/see-and-do/mathematics-winton-gallery>  **Bletchley Park**  Bletchley Park, once the top-secret home of the World War Two Codebreakers is now a vibrant heritage attraction. <https://www.bletchleypark.org.uk/> |

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| **Social Media and Websites**  A who’s who of who to follow on social media….. |
| **Numberphile** – “Video’s about numbers and stuff” – A huge collection of short videos hosted by top mathematicians, scientists and maths popularisers to discuss strange and wonderful applications of maths.  **Stand up maths** – Matt Parker is possibly the only person to hold the prestigious title of London Mathematical Society Popular Lecturer while simultaneously having a sold-out comedy show at the Edinburgh Festival Fringe, Matt is always keen to mix his two passions of mathematics and stand-up.  Others include Ted Talks, Mathologer, 3Blue1Brown, Vsauce |

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| **Things to buy ready for September** |
| **Textbooks – you will need these from the first lesson in September.**  Second hand is fine; you may be able to find a Y13 wanting to sell theirs on.  **We will be following the Pearson Edexcel course.** |
| **Calculator**  We **very strongly recommend** you purchase a **Casio graphical calculator**. Look on eBay for second hand calculators, although a new one is a good investment. The staff have the Fx-CG50 model, however the many students use a FX-9750. They both have the necessary functions, but the Fx-CG50 does a few extra things.  The vast majority of students use a Casio graphical calculator, as do the staff, and we will teach you to use it during the course.  Your calculator is only useful to you if you know how to use it. We will guide you through it in lessons, but you may want to have a look at some of these guides and activities.  [Resources Archive - Casio Calculators](https://education.casio.co.uk/all-resources/?featured=How%20To#filtered)  Scroll down the page and look at how to do various Calculations, Equations and Functions.  Ignore the sections that you don’t recognise, e.g. matrices and finance section. |

**Expanding brackets   
and simplifying expressions**

**A LEVEL LINKS**

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

* When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
* When you expand two linear expressions, each with two terms of the form *ax* + *b*, where *a*≠ 0 and *b*≠ 0, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

**Example 1** Expand 4(3*x* − 2)

|  |  |
| --- | --- |
| 4(3*x* − 2) = 12*x* − 8 | Multiply everything inside the bracket by the 4 outside the bracket |

**Example 2** Expand and simplify 3(*x* + 5) − 4(2*x* + 3)

|  |  |
| --- | --- |
| 3(*x* + 5) − 4(2*x* + 3)  = 3*x* + 15 − 8*x* – 12  = 3 − 5*x* | **1** Expand each set of brackets separately by multiplying (*x* + 5) by 3 and (2*x* + 3) by −4  **2** Simplify by collecting like terms: 3*x*− 8*x*= −5*x* and 15 − 12 = 3 |

**Example 3** Expand and simplify (*x* + 3)(*x* + 2)

|  |  |
| --- | --- |
| (*x* + 3)(*x* + 2)  = *x*(*x* + 2) + 3(*x* + 2)  = *x*2 + 2*x* + 3*x* + 6  = *x*2 + 5*x* + 6 | **1** Expand the brackets by multiplying (*x* + 2) by *x* and (*x* + 2) by 3  **2** Simplify by collecting like terms: 2*x*+ 3*x* = 5*x* |

**Example 4** Expand and simplify (*x* − 5)(2*x* + 3)

|  |  |
| --- | --- |
| (*x* − 5)(2*x* + 3)  = *x*(2*x* + 3) − 5(2*x* + 3)  = 2*x*2 + 3*x* − 10*x* − 15  = 2*x*2 − 7*x* − 15 | **1** Expand the brackets by multiplying (2*x* + 3) by *x* and (2*x* + 3) by −5  **2** Simplify by collecting like terms: 3*x*− 10*x* = −7*x* |

Practice

**1** Expand.

**Watch out!**

When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is ‘+’; if the signs are different the answer is ‘–’.

**a** 3(2*x* − 1) **b** −2(5*pq* + 4*q*2)

**c** −(3*xy* − 2*y*2)

**2** Expand and simplify.

**a** 7(3*x* + 5) + 6(2*x* – 8) **b** 8(5*p* – 2) – 3(4*p* + 9)

**c** 9(3*s* + 1) –5(6*s* – 10) **d** 2(4*x* – 3) – (3*x* + 5)

**3** Expand.

**a** 3*x*(4*x* + 8) **b** 4*k*(5*k*2 – 12)

**c** –2*h*(6*h*2 + 11*h* – 5) **d** –3*s*(4*s*2 – 7*s* + 2)

**4** Expand and simplify.

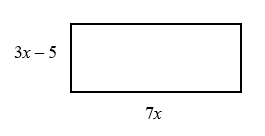
**a** 3(*y*2 – 8) – 4(*y*2 – 5) **b** 2*x*(*x* + 5) + 3*x*(*x* – 7)

**c** 4*p*(2*p* – 1) – 3*p*(5*p* – 2) **d** 3*b*(4*b* – 3) – *b*(6*b* – 9)

**5** Expand (2*y* – 8)

**6** Expand and simplify.

**a** 13 – 2(*m* + 7) **b** 5*p*(*p*2 + 6*p*) – 9*p*(2*p* – 3)

**7** The diagram shows a rectangle.

Write down an expression, in terms of *x*, for the area of the rectangle.

Show that the area of the rectangle can be written as 21*x*2– 35*x*

**8** Expand and simplify.

**a** (*x* + 4)(*x* + 5) **b** (*x* + 7)(*x* + 3)

**c** (*x* + 7)(*x* – 2) **d** (*x* + 5)(*x* – 5)

**e** (2*x* + 3)(*x* – 1) **f** (3*x* – 2)(2*x* + 1)

**g** (5*x* – 3)(2*x* – 5) **h** (3*x* – 2)(7 + 4*x*)

**i** (3*x* + 4*y*)(5*y* + 6*x*) **j** (*x* + 5)2

**k** (2*x* − 7)2 **l** (4*x* − 3*y*)2

Extend

**9** Expand and simplify (*x* + 3)² + (*x* − 4)²

**10** Expand and simplify.

**a**  **b** 

**Surds and rationalising the denominator**

**A LEVEL LINKS**

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

* A surd is the square root of a number that is not a square number,   
  for example  etc.
* Surds can be used to give the exact value for an answer.
* 
* 
* To rationalise the denominator means to remove the surd from the denominator of a fraction.
* To rationalise you multiply the numerator and denominator by the surd 
* To rationalise  you multiply the numerator and denominator by 

Examples

**Example 1** Simplify 

|  |  |
| --- | --- |
|  | **1** Choose two numbers that are factors of 50. One of the factors must be a square number  **2** Use the rule  **3** Use |

**Example 2** Simplify 

|  |  |
| --- | --- |
|  | **1** Simplify  and . Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number  **2** Use the rule  **3** Use  and  **4** Collect like terms |

**Example 3** Simplify 

|  |  |
| --- | --- |
| =  = 7 – 2  = 5 | **1** Expand the brackets. A common mistake here is to write  **2** Collect like terms: |

**Example 4** Rationalise 

|  |  |
| --- | --- |
| =  =  = | **1** Multiply the numerator and denominator by  **2** Use |

**Example 5** Rationalise and simplify 

|  |  |
| --- | --- |
| =  =  =  = | **1** Multiply the numerator and denominator by  **2** Simplify  in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number  **3** Use the rule  **4** Use  **5** Simplify the fraction:  simplifies to |

**Example 6** Rationalise and simplify 

|  |  |
| --- | --- |
| =  =  =  =  = | **1** Multiply the numerator and denominator by  **2** Expand the brackets  **3** Simplify the fraction  **4** Divide the numerator by −1  Remember to change the sign of all terms when dividing by −1 |

Practice

**1** Simplify.

**Hint**

One of the two numbers you choose at the start must be a square number.

**a**  **b** 

**c**  **d** 

**e**  **f** 

**g**  **h** 

**2** Simplify.

**Watch out!**

Check you have chosen the highest square number at the start.

**a**  **b** 

**c**  **d** 

**e  f** 

**3** Expand and simplify.

**a**  **b** 

**c**  **d** 

**4** Rationalise and simplify, if possible.

**a**  **b** 

**c**  **d** 

**e**  **f** 

**g**  **h** 

**5** Rationalise and simplify.

**a**  **b**  **c** 

# **Extend**

**6** Expand and simplify 

**7** Rationalise and simplify, if possible.

**a**  **b** 

**Rules of indices**

**A LEVEL LINKS**

**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

* *am* × *an* = *am* + *n*
* 
* (*am*)*n* = *amn*
* *a*0 = 1
*  i.e. the *n*th root of *a*
* 
* 
* The square root of a number produces two solutions, e.g. .

Examples

**Example 1** Evaluate 100

|  |  |
| --- | --- |
| 100 = 1 | Any value raised to the power of zero is equal to 1 |

**Example 2** Evaluate 

|  |  |
| --- | --- |
| = 3 | Use the rule |

**Example 3** Evaluate 

|  |  |
| --- | --- |
| =  = 9 | **1** Use the rule  **2** Use |

**Example 4** Evaluate 

|  |  |
| --- | --- |
|  | **1** Use the rule  **2** Use |

**Example 5** Simplify 

|  |  |
| --- | --- |
| = 3*x*3 | 6 ÷ 2 = 3 and use the rule  to give |

**Example 6** Simplify 

|  |  |
| --- | --- |
| = *x*8 − 4 = *x*4 | **1** Use the rule  **2** Use the rule |

**Example 7** Write  as a single power of *x*

|  |  |
| --- | --- |
|  | Use the rule , note that the fraction  remains unchanged |

**Example 8** Write  as a single power of *x*

|  |  |
| --- | --- |
|  | **1** Use the rule  **2** Use the rule |

Practice

**1** Evaluate.

**a** 140 **b** 30 **c** 50 **d** *x*0

**2** Evaluate.

**a**  **b**  **c**  **d** 

**3** Evaluate.

**a**  **b**  **c**  **d** 

**4** Evaluate.

**a** 5–2 **b** 4–3 **c** 2–5 **d** 6–2

**5** Simplify.

**a**  **b** 

**c**  **d** 

**Watch out!**

Remember that any value raised to the power of zero is 1. This is the rule *a*0 = 1.

**e**  **f** 

**g**  **h** 

**6** Evaluate.

**a**  **b**  **c** 

**d**  **e**  **f** 

**7** Write the following as a single power of *x*.

**a**  **b**  **c** 

**d**  **e**  **f** 

**8** Write the following without negative or fractional powers.

**a**  **b** *x*0 **c** 

**d**  **e**  **f** 

**9** Write the following in the form *axn*.

**a**  **b**  **c** 

**d**  **e**  **f** 3

# **Extend**

**10** Write as sums of powers of *x*.

**a**  **b**  **c** 

**Factorising expressions**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Factorising an expression is the opposite of expanding the brackets.
* A quadratic expression is in the form *ax*2 + *bx* + *c*, where *a* ≠ 0.
* To factorise a quadratic equation find two numbers whose sum is *b* and whose product is *ac*.
* An expression in the form *x*2 – *y*2 is called the difference of two squares. It factorises to (*x* – *y*)(*x* + *y*).

Examples

**Example 1** Factorise 15*x*2*y*3 + 9*x*4*y*

|  |  |
| --- | --- |
| 15*x*2*y*3 + 9*x*4*y* = 3*x*2*y*(5*y*2 + 3*x*2) | The highest common factor is 3*x*2*y*. So take 3*x*2*y* outside the brackets and then divide each term by 3*x*2*y* to find the terms in the brackets |

**Example 2** Factorise 4*x*2 – 25*y*2

|  |  |
| --- | --- |
| 4*x*2 – 25*y*2 = (2*x* + 5*y*)(2*x* − 5*y*) | This is the difference of two squares as the two terms can be written as (2*x*)2and (5*y*)2 |

**Example 3** Factorise *x*2 + 3*x* – 10

|  |  |
| --- | --- |
| *b* = 3, *ac* = −10  So *x*2 + 3*x* – 10 = *x*2 + 5*x* – 2*x* – 10  = *x*(*x* + 5) – 2(*x* + 5)  = (*x* + 5)(*x* – 2) | **1** Work out the two factors of *ac*= −10 which add to give *b*= 3  (5 and −2)  **2** Rewrite the *b* term (3*x*) using these two factors  **3** Factorise the first two terms and the last two terms  **4** (*x* + 5) is a factor of both terms |

**Example 4** Factorise 6*x*2 − 11*x* − 10

|  |  |
| --- | --- |
| *b* = −11, *ac* = −60  So  6*x*2 − 11*x* – 10 =6*x*2 − 15*x* + 4*x* – 10  = 3*x*(2*x* − 5) + 2(2*x* − 5)  = (2*x* – 5)(3*x* + 2) | **1** Work out the two factors of *ac*= −60 which add to give *b*= −11 (−15 and 4)  **2** Rewrite the *b* term (−11*x*) using these two factors  **3** Factorise the first two terms and the last two terms  **4** (2*x* − 5) is a factor of both terms |

**Example 5** Simplify 

|  |  |
| --- | --- |
| For the numerator:  *b* = −4, *ac* = −21  So  *x*2 − 4*x* – 21 = *x*2 − 7*x* + 3*x* – 21  = *x*(*x* − 7) + 3(*x* − 7)  = (*x* – 7)(*x* + 3)  For the denominator:  *b* = 9, *ac* = 18  So  2*x*2 + 9*x* + 9 = 2*x*2 + 6*x* + 3*x* + 9  = 2*x*(*x* + 3) + 3(*x* + 3)  = (*x* + 3)(2*x* + 3)  So    = | **1** Factorise the numerator and the denominator  **2** Work out the two factors of *ac*= −21 which add to give *b*= −4 (−7 and 3)  **3** Rewrite the *b* term (−4*x*) using these two factors  **4** Factorise the first two terms and the last two terms  **5** (*x* − 7) is a factor of both terms  **6** Work out the two factors of  *ac*= 18 which add to give *b*= 9  (6 and 3)  **7** Rewrite the *b* term (9*x*) using these two factors  **8** Factorise the first two terms and the last two terms  **9** (*x* + 3) is a factor of both terms  **10** (*x* + 3) is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1 |

Practice

**1** Factorise.

**Hint**

Take the highest common factor outside the bracket.

**a** 6*x*4*y*3 – 10*x*3*y*4 **b** 21*a*3*b*5 + 35*a*5*b*2

**c** 25*x*2*y*2 – 10*x*3*y*2 + 15*x*2*y*3

**2** Factorise

**a** *x*2 + 7*x* + 12 **b** *x*2 + 5*x* – 14

**c** *x*2 – 11*x* + 30 **d** *x*2 – 5*x* – 24

**e** *x*2 – 7*x* – 18 **f** *x*2 + *x* –20

**g** *x*2 – 3*x* – 40 **h** *x*2 + 3*x* – 28

**3** Factorise

**a** 36*x*2 – 49*y*2 **b** 4*x*2 – 81*y*2

**c** 18*a*2 – 200*b*2*c*2

**4** Factorise

**a** 2*x*2 + *x* –3 **b** 6*x*2 + 17*x* + 5

**c** 2*x*2 + 7*x* + 3 **d** 9*x*2 – 15*x* + 4

**e** 10*x*2 + 21*x* + 9 **f** 12*x*2 – 38*x* + 20

**5** Simplify the algebraic fractions.

**a**  **b** 

**c**  **d** 

**e**  **f** 

**6** Simplify

**a**  **b** 

**c**  **d** 

# **Extend**

**7** Simplify 

**8** Simplify 

Answers Expanding Brackets

**1 a** 6*x* – 3 **b** –10*pq* – 8*q*2

**c** –3*xy* + 2*y*2

**2 a** 21*x* + 35 + 12*x* – 48 = 33*x* – 13

**b** 40*p* – 16 – 12*p* – 27 = 28*p* – 43

**c** 27*s* + 9 – 30*s* + 50 = –3*s* + 59 = 59 – 3*s*

**d** 8*x* – 6 – 3*x* – 5 = 5*x* – 11

**3 a** 12*x*2 + 24*x* **b** 20*k*3 – 48*k*

**c** 10*h* – 12*h*3 – 22*h*2 **d** 21*s*2 – 21*s*3 – 6*s*

**4 a** –*y*2 – 4 **b** 5*x*2 – 11*x*

**c** 2*p* – 7*p*2 **d** 6*b*2

**5** *y* – 4

**6 a** –1 – 2*m* **b** 5*p*3 + 12*p*2 + 27*p*

**7** 7*x*(3*x* – 5) = 21*x*2 – 35*x*

**8 a** *x*2 + 9*x* + 20 **b** *x*2 + 10*x* + 21

**c** *x*2 + 5*x* – 14 **d** *x*2 – 25

**e** 2*x*2 + *x* – 3 **f** 6*x*2 – *x* – 2

**g** 10*x*2 – 31*x* + 15 **h** 12*x*2 + 13*x* – 14

**i** 18*x*2 + 39*xy* + 20*y*2 **j** *x*2 + 10*x* + 25

**k** 4*x*2 − 28*x* + 49 **l** 16*x*2 − 24*xy* + 9*y*2

**9** 2*x*2 − 2*x* + 25

**10 a**  **b** 

Answers Surds

**1 a**  **b** 

**c**  **d** 

**e**  **f** 

**g**  **h** 

**2 a**  **b** 

**c**  **d** 

**e**  **f** 

**3 a** −1 **b** 

**c**  **d** 

**4 a**  **b** 

**c**  **d** 

**e**  **f** 

**g**  **h** 

**5 a**  **b**  **c** 

**6** *x* − *y*

**7 a**  **b** 

Answers Indices

**1 a** 1 **b** 1 **c** 1 **d** 1

**2 a** 7 **b** 4 **c** 5 **d** 2

**3 a** 125 **b** 32 **c** 343 **d** 8

**4 a**  **b**  **c**  **d** 

**5 a**  **b** 5*x*2

**c** 3*x* **d** 

**e**  **f** *c*–3

**g** 2*x*6 **h** *x*

**6 a**  **b**  **c** 

**d**  **e**  **f** 

**7 a** *x*–1 **b** *x*–7 **c** 

**d**  **e**  **f** 

**8 a**  **b** 1 **c** 

**d**  **e**  **f** 

**9 a**  **b** 2*x*–3 **c** 

**d**  **e**  **f** 3*x*0

**10 a**  **b**  **c** 

Answers factorising

**1 a** 2*x*3*y*3(3*x* – 5*y*) **b** 7*a*3*b*2(3*b*3 + 5*a*2)

**c** 5*x*2*y*2(5 – 2*x* + 3*y*)

**2 a** (*x* + 3)(*x* + 4) **b** (*x* + 7)(*x* – 2)

**c** (*x* – 5)(*x* – 6) **d** (*x* – 8)(*x* + 3)

**e** (*x* – 9)(*x* + 2) **f** (*x* + 5)(*x* – 4)

**g** (*x* – 8)(*x* + 5) **h** (*x* + 7)(*x* – 4)

**3 a** (6*x* – 7*y*)(6*x* + 7*y*) **b** (2*x* – 9*y*)(2*x* + 9*y*)

**c** 2(3*a* – 10*bc*)(3*a* + 10*bc*)

**4 a** (*x* – 1)(2*x* + 3) **b** (3*x* + 1)(2*x* + 5)

**c** (2*x* + 1)(*x* + 3) **d** (3*x* – 1)(3*x* – 4)

**e** (5*x* + 3)(2*x* +3) **f** 2(3*x* – 2)(2*x* –5)

**5 a**  **b** 

**c**  **d** 

**e**  **f** 

**6 a**  **b** 

**c**  **d** 

**7** (*x* + 5)

**8** 